cision of such values as these, the average delicacy of female discrimination between the two points is to that of the male, in a ratio that lies somewhere between 7 to 6 and 8 to 7, or thereabouts. It will be recollected that the former ratio was that between the median female and the median male.

The *variability* of the discriminative power appears from the observations to be distinctly higher among females than among males. Measuring it in the usual way, by the half difference between the two quartiles, which is the same thing as the *probable error* of a single observation, or else by any multiple of this, as by the *mean* error, we find that the variability among females is to that among males as 3·70 to 3·25, say as 8 to 7. It is in consequence of this that so large a difference is shown between the relative sensitivity of the two sexes, at the right and left extremes of their respective curves in Fig. 2. We find from Table II. that the value of C—*c* at the 1st decile is 2·3 millimetres, and at the 9th decile it is only 0·7, the differences between the intermediate pairs decreasing regularly. The regularity of the decrease is not apparent in the actual observations, as shown in Fig. 1, nor in Table I., still there is nothing in what we see there that is incompatible with Fig. 2, while the fact of the difference between the right ends of the traces being much less than that between the left, is conspicuous.

Is it, however, a physiological fact that women are more variable than males in respect to discriminative touch, or are the observations affected by any extraneous cause of variability? I think that the recorded variability may in a very small part be accounted for by the fact that women vary much more than men in the exercise of sustained attention. Carelessness would affect the results in the same direction as diminished sensitivity. Thus suppose one part of a large number of persons who were all really alike in sensitivity, to be very careless, and the remainder to be scrupulously careful; the minds of the careless would be apt to wander; they would then fail to notice the first just-perceptible sense of doubleness, and would appear, in consequence, to be more obtuse than the careful ones. Though the range of variability was in reality *nil*, the existence of carelessness would introduce variability into the records. Some women are religiously painstaking, as much so as any men; but the frivolity of numerous girls, and their incapacity of, or unwillingness to give, serious attention, is certainly more marked than among men of similar ages. Women may, however, be really more variable than men in respect to sensitivity, because they seem more variable in a few other respects, such as in stature and obesity. Many more very tall girls are to be seen now-a-days among the upper classes than formerly, but the run of the statures among men has not altered quite so much. The multitude of extraordinarily obese women who used to frequent Vichy for the cure of fatness, were wonderful to behold; but they are no longer to be seen in their former abundance, as the fashion of treatment has changed within recent years. Again, it appears that women vary much more widely than men in respect of their morality; to which assertion I would quote Tennyson as a corroborative witness, who writes as follows, in Merlin's soliloquy on the character of Vivien:—

> "For men at most differ as heaven and earth,
> But women best and worst as heaven and hell."

Since Fig 2 is true to scale, it is easy to utilise it for ascertaining the class-place of any man or woman in respect to the form of sensitivity now in question. The whole process would be as follows:—Take a pair of compasses, and find with them by experiment the just-perceptible interval across on the nape of the neck of the person tested; then apply the compasses, to Fig. 2, keeping one (the lower) of its points always on the base line of the Fig., and holding the compasses so that the line joining its points shall be perpendicular to that base line. Slide the lower point of the compasses along the base line until the upper point touches the male or female trace, as the case may be; then read off the grade at which the lower point stands on the base line. Suppose it to be 35°; we thereby learn that 35 per cent. of the same sex have more sensitivity than the person tested, and that 65 per cent. have less. Similarly for any other value.

It would, I think, be well worth the while of an inquirer to repeat these tests, to revise my results, and to pursue the subject much further. If any one should feel disposed to do so, I would suggest that he should make his measurements with the cheap form of bow compasses, in common use by carpenters. The legs are connected not by a joint, but by a spring which tends to separate them, and they are brought together to any desired interval by turning a screw with the finger and thumb, which overcomes the spring. The interval between the points could easily be measured on a separate scale; all the more easily, if there were a slight depression at the zero point of the scale, in which one leg might be securely rested.

FRANCIS GALTON.

THE RELATION OF MATHEMATICS TO ENGINEERING.[1]

MATHEMATICS has been described in this room as a good servant but a bad master. It will be my duty this evening to prove by suitable illustration the first half of the proposition, and to show the service mathematics has rendered and can render to engineers and engineering.

In our charter the Institution of Civil Engineers is defined as "A society for the general advancement of Mechanical Science, and more particularly for promoting the acquisition of that species of knowledge which constitutes the profession of a Civil Engineer, being the art of directing the great sources of power in nature for the use and convenience of man, as the means of production and of traffic in states both for external and internal trade, as applied in the construction of roads, bridges, aqueducts, canals, river navigation and docks, for internal intercourse and exchange, and in the construction of ports, harbours, moles, breakwaters and lighthouses, and in the art of navigation by artificial power for the purposes of commerce, and in the construction and adaptation of machinery, and in the drainage of cities and towns." No better definition can, I think, be found for our profession than that it is the art of directing the great sources of power in nature for the use and convenience of man. It covers all that the widest view of our work can include, and it excludes those applied sciences, such as medicine, which deal with organised beings. Mathematics has to deal with all questions into which measurement of relative magnitude enters, with all questions of position in space, and of accurate determination of shape. Engineering is a mathematical science in a peculiar sense. Medicine, the other great profession of applied science, has but little to do with questions of measurement of magnitudes, or of geometry; but the engineer finds them enter into everything with which he has to deal, and enter in the most diverse ways. The thing he has to determine is that the means he employs is enough and not unnecessarily more than enough to attain the end in view. For this he must numerically measure the end and the means and see that they are justly proportioned to each other. It is useless this evening to waste time proving, what all will admit, that no one can be even the humblest engineer without a knowledge of arithmetic and enough of geometry to enable him to read a drawing, that some trigonometry, some rational mechanics and a knowledge of projections, is a very useful part of the mental equipment of a draughtsman. It is hardly necessary to call attention to the great economy in the labour of calculations effected by the use of logarithms, a mathematical instrument for which we are indebted to Napier. We may with more profit examine what use the higher mathematics can be to the practical engineer, and what has been done in the past for engineering by its aid.

Judging from etymology, mathematics must have been begun by engineers; for surely geometry is the work of the earth measurer or land surveyor. But since the prehistoric times when geometry was initiated, engineers have not added much that is new to mathematics. They have rather sought amidst the stores of the mathematician and selected the handiest mathematical tool they could find for the particular purpose of the moment, but have done little or nothing in return in the way of improving the tools which they borrow. In this respect the relation of engineering to mathematics differs much from its relation to experimental physics. In electricity, magnetism and heat, engineers have from their practical experience repeatedly corrected the ideas of the theorists, and have started the science on more accurate lines. If our subject to-night had been the use of the practical applied science of engineering in promoting the development of pure mathematics, we should speedily find that there was hardly any material for discussion. The account being all on one side, let us see to what the debt of the engineer to the mathematician amounts.

There is no department of practical engineering in which the application of mathematics is more familiar than in that which relates to the calculation of the strength and rigidity of structures of various kinds. It is impossible to take up any book dealing with the subject without finding that it is crammed either with mathematical formulæ, or with geometrical figures. The question is not whether mathematics is necessary to an adequate comprehension of the subject, but whether analytical or purely geometrical methods are more convenient. Of course one might occupy many lectures in discussing the practical application of mathematics to the question of bridge building, roofs, guns, shafting, and the like. Our object must be to illustrate by various examples rather than to attempt anything like a complete discussion.

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[1] The "James Forrest" Lecture, delivered at the Institution of Civil Engineers, by Dr. John Hopkinson, F.R.S., on May 3.

Consider the case of a long strut, so long that its transverse dimensions can be regarded as insignificant in comparison with its length. Whilst the strut remains perfectly symmetrical about its middle line, its strength will depend only upon the resistance of the material to crushing. Everyone knows that this would be an inadequate conclusion; we have to consider another element, namely, its stability, that is, we must examine what will happen to the strut if from any cause it is displaced somewhat from the direct line between its extremities. A mathematical discussion of the question results in a differential equation of the second order with one independent variable. Upon consideration of this we are enabled to see that if the thrust upon the strut be less than a certain critical value, a slightly bent strut will tend to return to its straight condition; but that if the thrust upon the strut be greater than this critical value the displacement will tend to increase, and the strut will give way. Further, that the critical value will depend upon whether one or both of the two extremities of the strut are held free, or whether they are rigidly attached by flanges or otherwise, so that the direction of the axis of the strut at this point must remain unaltered. Again, we infer that if the ends are held rigidly fixed, the length of the strut may be twice as great for a given critical value of the thrust as if the two ends are free to turn. We can also infer what the critical value will be for struts of various lengths and of varying cross sections. This critical value depends not upon the resistance of the material to crushing, but upon its rigidity.

Another example, having a certain degree of similarity with the case of struts, is that of a shaft running at a high number of revolutions per minute, and with a substantial distance between its bearings; for simplicity, we will suppose that there are no additional weights, such as pulleys, upon the shaft. How will the shaft behave itself in regard to centrifugal force as the speed increases? In this case, so long as the shaft remains absolutely straight it will not tend to be in any way affected by the centrifugal force, but suppose the shaft becomes slightly bent, it is obvious to anyone that if the speed be enormously high this bending will increase, and go on increasing until the shaft breaks. In this case also we may use mathematical treatment; we find that the condition of the shaft is expressed by a differential equation of the fourth order, and from consideration of the solution of this equation we can say that if the speed of any particular shaft be less than a certain critical speed, the shaft will tend to straighten itself if it be momentarily bent, but that, on the other hand, if the speed exceeds this critical value, the bending will tend to increase with the probable destruction of the shaft. I do not know that either of these two questions can be properly understood without some knowledge of differential equations.

A problem having a certain analogy to those to which I have just referred is that of hollow cylinders under compression from without, such as boiler tubes. Whether the tubes be thick or thin, so long as they are perfect circular cylinders, they should stand until the material was crushed. But if the tubes are thin, what will happen if the tube from any cause deviate ever so little from the cylindrical form? The solution cannot be obtained without a substantial quantity of mathematics.

The next illustration shows how a mathematical conclusion, correct within the limits to which it applies, may mislead if applied beyond those limits, and how a more thorough mathematical discussion will give a correct result. Considering a case of shafting in torsion it was shown by Coulomb that the stiffness and strength of a shaft having the form of a complete circular cylinder could be readily calculated if the transverse elasticity of the material and its resistance to shearing were known. From the complete symmetry about the axis it is evident that points which lie in a plane perpendicular to the axis before twisting will still be in that plane when the shaft is twisted; it is also clear that the angle through which all points in the same plane move will be the same; hence the problem was as simple as problem could be. But many who had occasion to make use of Coulomb's results gave them an application which was wholly unwarranted. They assumed that they were equally applicable to other cases than complete circular cylinders; they assumed in fact that every point of the material which lay in a plane perpendicular to the axis would remain in that plane when the shaft was twisted, whether the shaft was symmetrical about its axis or not, and they consequently arrived at very erroneous results. That the assumption was erroneous is obvious enough from a consideration of an extreme case. In Fig. 1 is shown in cross-section a hollow cylindrical shaft, which is not complete, but divided by a plane passing through its axis. In this case the shaft when twisted will be as illustrated in the side elevation; two points, A and B, were in one plane perpendicular to the axis when the shaft was free from twist; they cease to be in one plane when the shaft is twisted. St. Venant[1] in 1855 investigated the question of shafts without making incorrect assumptions; he expressed the condition of the material by a partial differential equation of the second order, and gave suitable surface conditions. A general solution of the problem for all forms of shafts has not been obtained, but St. Venant gives a number of solutions for particular forms, and he obtains some general results of interest. In all cases the stiffness of the shaft is less than would be inferred from an erroneous application of Coulomb's theory. Fig. 2 shows diagrammatically the strain in a shaft of triangular section; the full lines indicate that the parts of the shaft which lay in one plane before twisting when twisted rise above the plane; the dotted lines indicate that they lie behind the plane of the paper. The shearing stress is least at the angles of the triangle, and is greatest at the middles of the sides. At this point then the shaft will begin to break under torsion. The fact is probably well known to men of practical experience, but it is directly contradictory to the conclusion at which one would arrive by a careless use of Coulomb's theory beyond the narrow limits within which it is applicable. The longitudinal ribs which one often sees on old cast-iron shafts are useful enough to give stiffness to the shafts against bending, but are good for very little if torsional stiffness or torsional strength is desired.

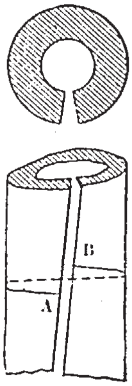

FIG. 1.

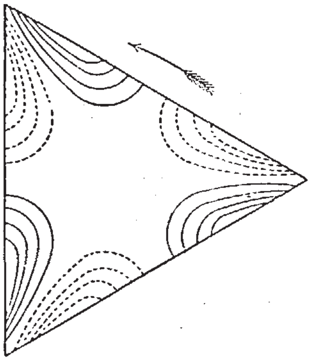

FIG. 2.

Another application of mathematical theory which has been carried somewhat further than the premises warrant is found in the case of girders. It is almost invariably customary to treat a girder as though the sections retain when the girder is bent the form and size which they had before bending. Making this assumption, it is very easy to calculate the strength and stiffness of a girder of any section. Unfortunately, the assumption is untrue; but, fortunately, it is approximately true in the case of most girders with which engineers in practice have to deal. That it is untrue can be readily seen from consideration of a girder of exaggerated form, the section of which is shown in Fig. 3. Any practical man would at once see that the outer parts of the flanges would add little to the strength of the girder, but according to the usual mathematical theory the outer parts of this flange should be as useful as the parts which are nearer to the web. This problem St. Venant also deals with in a rigid mathematical manner. Amongst other things, he showed that a girder of rectangular section, such as shown in Fig. 4, would, when bent, take the form shown by the curved lines in the same Fig. The last two examples show how a little knowledge may be a dangerous thing, and how easy it is for anyone who attempts to apply mathematics without adequate mathematical knowledge to be misled.

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[1] "Mémoires des Savants Étrangers," 1855; and Thomson and Tait, "Treatise on Natural Philosophy."

The theory of thick cylinders under bursting stress from within has many important practical applications to hydraulic presses and to guns. It has been discussed more than once in this room. As usual in considering these cases we are immediately led to differential equations which here are fortunately solved without serious difficulty, and the solution tells us the whole story. We learn that doubling the thickness by no means doubles the strength of the cylinder. And as a converse, that doubling the strength of the material will permit the thickness to be diminished to much less than one-half. Twenty-five years ago hydraulic presses were mostly made of cast-iron. Many people were not a little astonished at the great reduction in thickness and weight which became possible when steel was substituted for the weaker material. In the case of guns it is well known that greater strength can be obtained if the outer hoops are shrunk on to the inner ones.[1] Mathematical theory tells us what amount of shrinkage should give the best results. It may possibly not be worth while to follow the results of theory precisely, but without the guidance of theory it would not be unnatural to give so great a shrinkage that the gun would be weaker than if no shrinkage were used.

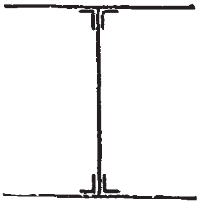

Fig. 3.

The rolling of ships in a seaway gives an illustration of a principle which has very varied application in many branches of physics. Suppose a body is capable of oscillating in a certain periodic time, and that it is submitted to a disturbing force of given period, the equation of motion easily shows that the resulting disturbance will be great if the two periods are equal or nearly equal. We meet with the principle in acoustics as resonance. If two tuning-forks are tuned to the same pitch, and one is sounded in the neighbourhood of the other, that other will presently be thrown into vibration by the waves transmitted through the air from the first. You may try a similar experiment at any time on any piano. Strike the higher G in the treble, the sound ceases on raising the finger. Now hold down the middle C, and again strike G; the C string at once takes up the note sounded, and can be heard after the exciting string has been silenced by damping. The same fundamental idea is found in the lunar theory in the term in the equation known as the evection, and again in the theory of Jupiter's satellites. The reason why the metals present in the solar atmosphere give black lines in the spectrum by absorption, corresponding in position with the bright lines in the spectrum which the same metals give when incandescent, is again the same. Gas will absorb or take in from the ether waves of the exact period which it is capable of giving to the ether. The general explanation of all these phenomena is easy. Imagine a pendulum, and suppose it experiences a periodic disturbing force, the first impulse of the disturbing force gives the pendulum a slight swing; the effect of the second impulse depends entirely on when it occurs; it may occur so as to neutralise the effect of the first, or it may occur so as to increase it. If the period of the force is the same as the natural period of the pendulum, the effect of the second, third, and later impulses will be added to the effect of the first, and the final disturbance will be great, even though the individual impulses be minute. But the mathematical theory tells us much more than any general explanation can do. It tells us exactly what the character of the effect will be, and its amount if the periods are nearly but not exactly the same. It tells us, too, exactly how friction affects the results. And the beauty of it is that the mathematical theory is much the same in all cases, so that having learned to deal with one case we are enlightened as to a host of others. The oscillating body may be an ironclad, or it may be an atom of hydrogen; the disturbing periodic force may be the waves of the Atlantic, or it may be the waves in the ether occurring five hundred millions of millions of times in a second; it is all one to the mathematician; the treatment is substantially the same.

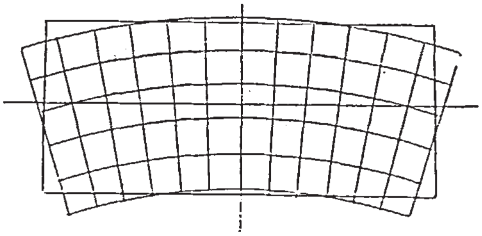

Fig. 4.

The question of the speed of ships and the power to propel them is probably more effectually treated by experiment on models, as was done by the late Mr. Froude, than by mathematics alone; but in order to learn from the experiments all they are capable of teaching, a mathematical understanding is needed. Given that we know by experiment all about a given model, that we know what force is needed to propel it at every speed, we want to know from these experiments how a great ship, 100 times as big, but similar in form in every respect, will behave; and here mathematics come in to aid us in making the inference.

The construction of ships at once leads us on to the methods of navigating them. In navigation I should find much material for my purpose, but navigation is not usually included in engineering, but many of the implements of navigation undoubtedly are. The mariners' compass has for ages been the mainstay of the navigator, and a simple enough instrument it was till it was disturbed by the iron of which ships came to be built. The disturbance of the compass by the iron of the ship was first seriously attacked by two senior wranglers, Sir G. Airy and Mr. Archibald Smith. The disturbance may be divided into two parts, the first due to the permanent magnetism of the ship, the second to the temporary magnetism induced by the earth's inductive action on the iron of the ship—the first causes the semicircular, the second the quadrantal, error. One has only to open the "Admiralty Manual of Deviations of the Compass" to see how the mathematics of Archibald Smith have accomplished a proper understanding of the subject. The errors of the compass are dealt with in two ways: they are compensated by soft iron correctors, and by permanent magnets so placed as to have an effect equal and opposite to the effect of the temporary and permanent magnetism of the ship. Or they are dealt with by formulæ of correction which enable the error to be calculated when the course of the ship and the conditions of the earth's magnetism are given, or a combination of the two methods is used. Either method is based on Archibald Smith's theory. It is not possible to leave the subject of the mariners' compass without referring to the great improvements of Lord Kelvin. The improvements relate to every part of the instrument, and I venture to say that none of them could have been made by anyone but a mathematician. In order to get his card steady he knew that its period must be different to any possible period of the waves, or he would have the resonance to which I have just referred coming in, so he gave his card a considerable moment of inertia; but this was managed with a light card so that small needles could be used. If the needles are small the correction by soft iron masses and by permanent magnets is easier and more accurate. Then the bowl of the compass had to be suitably carried so that it would not be unduly disturbed by shock, and provision had to be made for damping by fluid friction the oscillations of the bowl if they occurred. Lastly, a most beautiful method of correcting the compass, without taking a sight, was discovered. In every detailed improvement one can detect that the inventing mind was that of a most able and trained mathematician.

An essential of safe navigation is an efficient system of lighthouses. The optical problem of the lighthouse engineer is to construct apparatus which shall usefully direct all the light produced. The present forms of apparatus are in their leading features due to Fresnel, the able mathematician, who established on an absolutely firm foundation the undulatory theory of light. To properly design an optical apparatus formulæ must be used, and the advantage is great if the designer can with ease manufacture the formulæ he requires.

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[1] Lamé, "Leçons sur la Théorie Mathématique de l'Élasticité des corps solides." 14th Leçon.

Submarine telegraphy yields some interesting examples of the application of the higher mathematics. When a cable across the Atlantic was first seriously entertained, the first point to be settled was, how many words a minute could be sent through such a cable. This was the most practical question possible. Upon the answer depended the prospect of the cable paying commercially if successfully laid. The matter was dealt with by Prof. Thomson,[1] of Glasgow, now Lord Kelvin. He showed that the propagation of an electric disturbance in a cable could be expressed by a partial differential equation, and that the solution of this equation under certain conditions applicable to practice could be expressed either by a definite integral or by an infinite series. The values of these were calculated, and hence before an Atlantic cable was laid at all it was known how long it would take a signal to reach the opposite shore, and how much its intensity would be diminished in transmission. Referring to Fig. 5, abscissæ represent time, reckoned from the time of making contact at the sending end of the cable, ordinates the currents at the receiving ends, curve (1) gives these currents when the contact at the receiving end, after being made, is continuously maintained. It will be observed that for a time a there is hardly any current at the receiving end, that then the current rapidly increases and attains to half its final value after a time equal to about $5a$. Curves (1) . . . . (7) show the currents at the receiving ends when the contact is made at the sending end maintained for times a, $2a$ . . . $7a$ respectively, and then broken. Looking at curve (1) one sees how small is the amount of current and how long it lasts compared with the time during which contact is made. The time a depends on the length and character of the cable; it is equal to $k\,c\,l^2 \log_e \frac{4}{3}/\pi^2$, where k is the resistance per unit length, c the capacity per unit length, and l the length of the cable. The knowledge of what is the commercial value of a cable depends on a knowledge of the value of a, and this cannot be obtained without knowing the differential equation $c\,k\frac{dv}{dt}=\frac{d^2v}{dx^2}$,[2] to which I have referred, and its by no means simple solution either as a definite integral or as an infinite series. So far as I know, this piece of higher mathematics cannot be evaded by any mere elementary treatment. The transmission of disturbance in a cable is quite different from the transmission of sound waves in air, which move with constant velocity. If the cable be doubled in length, it takes four times as long for the signal to pass through it instead of just twice as long, as would be the case if it were a proper wave motion. In fact the time of passage between the making of contact at the sending end of the cable and the beginning of the resulting disturbance at the receiving end, varies as the square of the length of the cable. The mathematical theory is exactly the same as that of the transmission of heat in a plate, one surface of which is suddenly exposed to a temperature different to the temperature of the plate. This is constantly occurring in the application of mathematics—one piece of mathematical work serves for many physical problems having apparently little in common. Fourier long ago discussed the heat problem, little dreaming that his analysis would be just what was wanted for ascertaining how fast signals could be sent across the Atlantic by a system of telegraphy which in his days had not even been projected in its simplest form. The same differential equation also gives the theory of the transmission of telephonic messages through cables; but the solution is then easier, and tells us exactly why it is so much more difficult to speak through 100 miles of cable than through 1000 miles of overhead line. As I have just stated, the differential equation of the disturbance in the cable is $c\,k\frac{dv}{dt}=\frac{d^2v}{dx^2}$. A musical note of period T spoken into the cable through a telephone is properly represented by $A\sin\frac{2\pi t}{T}$; the disturbance in the cable will be—

$$v = A\,e^{-x\sqrt{kc\pi/T}}\sin\left(\frac{2\pi t}{T}-x\sqrt{kc\pi/T}\right),$$

as may be easily verified by differentiating. This equation tells us everything. It tells us the rate at which the waves diminish with the distance. This rate increases with the resistance, with the capacity and with the frequency. If the capacity is at all considerable the diminution is rapid. The velocity of the waves is not the same for all frequencies, as is the case with waves in air, but varies as the square root of the period, so that if two notes were sounded the high note would arrive after the low notes, and the resultant effect would be entirely destroyed. Here, again, it is difficult to see how the differential equation and its solution can be evaded.

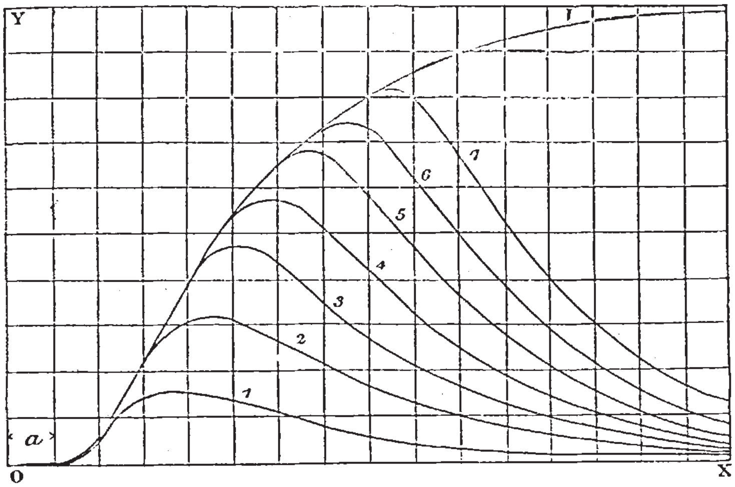

FIG. 5.

Though the history of the telegraph dates only from a little more than fifty years ago, it is ancient in comparison with the other great applications of electrical science, which have received their development during the last fifteen years. Here again mathematics which are not quite elementary have played their part. In the theory of transformers we find another illustration of the need of knowing how formulæ are obtained if they are to be correctly applied. The early transformers were made with unclosed magnetic circuits; there was an iron core, but the lines of magnetic force passed through air for a considerable part of their path. In this case a complete mathematical theory was not very difficult. But speedily closed magnetic circuits were found to be better, and the relation of magnetic induction and magnetic force became all important. If anyone were to apply mathematical formulæ, which were true for the earlier transformers, to the later ones, his results would be inaccurate. Indeed a wholly different method of attack on the problem was needed, taking account of the facts as they are, and not applying results which were true of older apparatus to cases essentially distinct.[1]

The employment of alternating currents has brought into use, as a necessity for understanding the actually observed phenomena, a great deal of mathematics. Why is the apparent resistance of a conductor greater for an alternating current than for a direct current? And by resistance I do not mean the quasi-resistance due to self-induction.[2] The mathematical electrical theory is ready with an answer; it is ready, too, to tell us how the difference depends upon the frequency of the current and on the size of the conductor. In the case of a cylindrical conductor the solution involves a knowledge of Bessel's functions. We learn that if the current has a high frequency, or if the conductor be large, there will be very little current in the centre of the cylinder, and that therefore for any practical purpose the centre of the cylinder might just as well not be there; the current is largely confined to the part of the conductor near to its surface. The currents at different depths in the conductor attain to their maximum values at different times; those near the surface of the cylinder occur before those at some distance from the surface. The mathematical conditions are expressed by the same equation as is used to express the disposition of heat in a cylinder the surface of which is submitted to a periodic variation of temperature. Anyone who had thoroughly mastered the heat problem would be quite prepared to deal with the problem of currents in a conductor. It cannot be too often repeated, any piece of pure mathematics which finds one application to a physical problem is almost sure to find, in exactly the same form, applications to other problems which superficially are absolutely distinct. The differential equation in this case is $kc\frac{dv}{dt} = \left(\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr}\right)$, the similarity of physical condition to the problem of linear propagation of heat is close, but the mathematics differ materially owing to the presence of the term $\frac{1}{r}\frac{dv}{dr}$ in the equation. Mathematics deals with the relation of quantities to each other without troubling as to what the physical meaning of the quantities may be. Hence it is that the mathematical treatment of two such problems as the distribution of currents in a cylindrical conductor and of heat in a cylinder is identical, whereas the treatment of the distribution of heat in a cylinder is quite distinct from the treatment of the distribution of heat in a sphere or in a solid bounded by two parallel planes.

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[1] "Mathematical and Physical Papers," vol. ii. p. 61. Sir W. Thomson.

[2] v is the potential, t the time, and x the distance from the sending end of the cable.

[1] *Proceedings* of Royal Society, February 17, 1887.

[2] Lord Rayleigh, *Phil. Mag.*, vol. xxi. p. 381.

A curious phenomenon was observed in the large alternate-current machines at Deptford when connected to the long cables intended to take the current to London. The pressure at the machines when connected to the conductors was, under certain conditions, actually greater than when not so connected. The phenomenon is one of resonance very analogous to the heavy rolling of ships when the natural period of roll is about the same as the period of the waves.[1] The period of the alternating current corresponds to the period of the waves, the self-induction of the machine to the moment of inertia of the ship, the reciprocal of the capacity to the stiffness of the ship, and the electrical resistance of the conductors to the frictional resistance to rolling. The mathematics in the two cases is then the same. The effect was predicted long before it was observed in a form calculated to cause trouble.

A problem which is still agitating electrical engineers is that of running more than one alternate-circuit dynamo machine connected to the same system of mains. Before the matter became one of practical concern, it was considered in this room, and it was shown mathematically that it was possible to run independently-driven alternators in parallel but impossible to run them in series. That is to say, that if two alternators were connected to the same mains they would tend to adjust themselves in relation to each other so that their currents could be added, but that if an attempt were made to couple them, so that their pressures should be added, they would adjust themselves so that their effects would be opposed.[2]

Perhaps of all engineering problems which have received their solution in the last hundred years that of the greatest practical importance is the conversion of the energy of heat into the energy of visible mechanical motion. The science of thermodynamics has advanced along with the practical improvement of the steam-engine. By its aid, particularly by the aid of the so-called second law, we know what is possible of attainment by the engineer under given conditions of temperature. I must not trench on the subject of one of my successors, but I may point out that our knowledge of the second law of thermodynamics was first developed by means of mathematics, and that to-day its neatest expression is by means of partial differential coefficients. The two most notable names in connection with the development of the second law of thermodynamics in harmony with the first are those of Kelvin and Clausius; both dealt with the subject in a mathematical form not comprehensible to those who have not had substantial mathematical training.

Illustrations such as these might be multiplied almost indefinitely. They show that the advancement of the science of engineering has been aided in no inconsiderable measure by the labours of mathematicians directly applying the higher mathematical methods to engineering problems. They show, too, one way in which respect for a formula may be dangerous, one way in which it is true that mathematics may be a bad master. In St. Venant's problems we have an example in which the use of older results of limited application in cases where the assumptions on which they rest are not true will mislead. The examples show the proper remedy; it is a more complete application of mathematical methods. The error is just one which a man will make who has the power to use a formula without a ready understanding of how it is arrived at. A practical man, ignoring mathematical results, might or might not escape the error of supposing that a triangular shaft would break at the angles under torsion; the half-educated mathematician would certainly fall into the snare from which complete mathematical knowledge would deliver him. You can only secure the services of that good servant, mathematics, and escape the tyranny of a bad master by thoroughly mastering the branches of mathematics you use. The mistake caused by the wrong application of mathematical formulæ is only to be cured by a more abundant supply of more powerful mathematics.

There is another drawback to the use of results, taken, it may be, out of an engineering pocket-book by those who are not prepared to understand how they are reached and on what foundations they rest. The educational advantage is lost. The close observation which enabled the earlier engineers to proportion their means to the ends to be attained was no doubt very laborious, and the results could not be applied to cases much different from those which had been previously seen, but the effect on the character of the engineer was great. In like manner, to thoroughly understand the theory of an engineering problem makes a man able to understand other problems, and in addition to this precisely the same mathematical reasoning applies to many cases. The mere unintelligent use of a formula loses all this; it leaves the mind of the user unimproved, and it gives no help in dealing with questions similar in form though different in substance.

But even the use of mathematics by competent mathematicians is not without drawbacks. Mathematical treatment of any problem is always analytical—analytical, I mean, in this sense that attention is concentrated on certain facts, and other facts are neglected for the moment. For example, in dealing with the thermodynamics of a steam-engine, one dismisses from consideration very vital points essential to the successful working of the engine, questions of strength of parts, lubrication, convenience for repairs. But if an engineer is to succeed he must not fail to consider every element necessary to success; he must have a practical instinct which will tell him whether the instrument as a whole will succeed. His mind must not be only analytical, or he will be in danger of solving bits of the problems which his work presents, and of falling into fatal mistakes on points which he has omitted to consider, and which the plainest, intelligent practical man would avoid almost without knowing it.

Again, the powers of the strongest mathematician being limited, there is a constant temptation to fit the facts to suit the mathematics, and to assume that the conclusions will have greater accuracy than the premises from which they are deduced. This is a trouble one meets with in other applications of mathematics to experimental science. In order to make the subject amenable to treatment, one finds, for example, in the science of magnetism, that it is boldly assumed that the magnetisation of magnetisable material is proportional to the magnetising force, and the ratio has a name given to it, and conclusions are drawn from the assumption, but the fact is, no such proportionality exists, and all conclusions resulting from the assumption are so far invalid. Wherever possible, mathematical deductions should be frequently verified by reference to observation or experiment, for the very simple reason that they are only deductions, and the premises from which the deductions are made may be inaccurate or may be incomplete. We must always remember that we cannot get more out of the mathematical mill than we put into it, though we may get it in a form infinitely more useful for our purpose.

Engineers no doubt regard their profession from very different points of view; some think it a mere means of making money; some regard it as an instrumentality for benefiting the race; whilst others again delight in it as an interest in itself, and delight in it most of all when new knowledge is added to that which we know already. It is just the same with the medical profession; some attend patients for the guineas they receive, some give a very high place to motives of benevolence, whilst others love it as a field where new knowledge may be found and the delight of discovery enjoyed. In regard to the first class of engineers, I have no doubt a little skill in managing a board of directors or impressing a committee of Parliament will be much more useful to the engineer than a great deal of mathematics. Let him manage his board and buy his mathematician, and it is very probable he will make much more money than the mathematician or any other person of skill whom he may employ. But we cannot all of us make money in this way. In the future it is likely that educated men will have to work harder and receive less, and it is a great thing if their work can be made itself a joy, and surely this can best be by a thorough understanding of the reason of all they do by the feeling that they have full competence to form their own judgments without depending much on the authority of others. This can only be in the words of Sir John Herschel by a "sound and sufficient knowledge of mathematics, the great instrument of all exact inquiry, without which no man can ever make such advance in any of the higher departments of science as can entitle him to form an independent opinion on any subject of discussion within their range."

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[1] Institution of Electrical Engineers, November 13, 1884.

[2] Minutes of *Proceedings* Inst. C.E., April 5, 1883; Institution of Electrical Engineers, November 13, 1884.

After all, in any department of applied or pure science the highest satisfaction comes from accomplishing that which no one has done before, from disclosing what no one hitherto has known. If a department of the arts or sciences ceases to advance and becomes simply the application in known ways of known principles to obtain known ends, that department has lost its charm till the time comes for a fresh advent of change and development. To effect such advances it is easy to show that mathematics is a most necessary instrument. Here it is no drawback that the mind of the discoverer is too analytical; he may deal at his pleasure with one aspect of a problem, and it does not detract in any way from the value of his solution that he does not touch on incidental matters. Some of you who love the interest of continual advance in our science and practice, may look forward with a shade of sadness to a possible time when all is done or known which can be done or known, and the work of the engineer shall be merely applying principles discovered by his predecessors. In such a state, when the experience of the older generations shall control the practice of to day, the free use of mathematical methods may be effectually superseded by the application according to rule of mathematical formulæ. But it would be a much less interesting condition than the constant change of to-day, when the practical experience of ten years ago is in many departments rendered worthless by later discoveries. But we need not fear that such a time of petrifaction will come so long as, whilst reverencing the discoverers who have added to our knowledge, we endeavour to replace their methods by better, and expect that those who come after us will, in their time, improve upon ours. Our knowledge must always be limited, but the knowable is limitless. The greater the sphere of our knowledge the greater the surface of contact with our infinite ignorance.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

OXFORD.—By a resolution of Congregation, the care of such portions of, as may seem desirable, the Lee collection of anatomical and physiological specimens, which have for many years past been placed in the University Museum under care of the Linacre Professor, may be transferred to the care of other Professors with the sanction of the Trustees of the Lee bequest.

The date of the preliminary examinations in natural science in the Michaelmas term has been fixed for the Monday in the eighth week in Full Term in lieu of the dates hitherto observed.

The resolutions proposed by the committee on the granting of degrees for research were brought before Convocation on Tuesday afternoon. The first resolution, affirming the general principle that it was desirable that such degrees should be established, was agreed to *nemine contradicente*. Some difference of opinion manifested itself in the discussion on the succeeding resolutions. That which suggested that the new degrees should be styled Master of Science and Master of Letters was rejected on a division, with the intention that the titles should be further considered by the committee. The remainder of the resolutions were agreed to, and the drafting of a statute embodying the recommendations of the committee was referred to a committee consisting of the Vice-Chancellor and twelve others.

The annual Boyle Lecture was delivered on Tuesday evening by Prof. A. MacAlister, who chose as his subject, "Some morphological lessons taught by human variations." The lecture, which is held under the auspices of the Junior Scientific Club, was largely attended.

CAMBRIDGE.—The new Engineering Laboratory, just completed under the supervision of Prof. Ewing, will be formally opened on May 15, at 3 p.m., by Lord Kelvin, President of the Royal Society. The Vice-Chancellor will preside, and it is expected that Prof. Kennedy and others will deliver addresses on the occasion. After the opening ceremony Prof. and Mrs. Ewing will be "at home" in the laboratory, in order to give members of the University an opportunity of seeing for themselves the provision that has been made for the scientific study of engineering.

The degree of Doctor of Laws will, on May 10, be conferred upon Dr. Carl Theodor von Inama-Sternegg, honorary Professor of Political Science in the University of Vienna, and President of the KK. Statistical Central-Commission of the Austro-Hungarian Empire. Prof. von Inama-Sternegg was President of the Demographic Section of the International Congress of Hygiene held in London in 1891, but was unable to visit Cambridge with the other members of the Congress for the purpose of receiving the degree.

At the same Congregation the complete degree of M.A. will be conferred on Prof. Ewing's able demonstrators, Mr. W. E. Dalby and Mr. C. G. Lamb, who are already Bachelors of Science of the University of London.

Mr. Oscar Browning, who is an officier d'Académie, will next month represent the University of Cambridge at the festal opening of the new Palais des Facultés of the Académie of Caen.

Dr. Hobson, F.R.S., has been appointed a syndic of the library, in the room of the late Prof. Robertson Smith.

A grant of £20 from the University chest has been made to Mr. H. Yule Oldham, University Lecturer in Geography, for maps and apparatus.

The growth of the botanical department under the direction of Deputy-Prof. F. Darwin, F.R.S., has led the General Board of Studies to recommend that his stipend as reader, and that of Mr. W. Gardiner, F.R.S., as lecturer, should be increased to £150 a year. An additional demonstratorship in botany is also proposed. The Board further recommend that the annual stipend of Mr. S. J. Hickson, as lecturer in Advanced Morphology, should be increased to £100.

The Natural Sciences Tripos, for which there are about 130 candidates, begins on May 23, and will extend to June 12.

THE Convocation of the University of London met on Tuesday. It was expected that a warm discussion would take place on the Gresham scheme, but the expectation was not realised, as the chairman, Mr. E. H. Busk, ruled out of order all motions relating to that subject. One of these resolutions, standing in the name of Mr. Thiselton-Dyer, was—"That Convocation, while reserving its right to represent its views before the proposed Statutory Commission, hereby expresses its general approval of the Report of the Royal Commission." The *Times* reports that, when this and other motions had been ruled out of order, Prof. Silvanus Thompson moved the adjournment of the House. After some discussion an amendment to Prof. Thompson's motion—that the House should adjourn until seven o'clock—was accepted almost unanimously, the common object of all parties being to ascertain the result of the voting for the annual committee, for the election of which the two parties had their separate lists, one list consisting of those who were practically in favour of the Gresham scheme, and the other desiring to leave the whole question in the hands of the joint committee. The former party carried their whole list.

On the adjournment before the declaration of the poll a body of some 230 graduates met in the Graduates'-room of the University to draw up a protest against their having been again prevented from discussing the report of the Gresham Commission. Sir Henry Roscoe presided, and the speakers were Mr.